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Article

Network reliability evaluation and management science applications in resource allocation for police organizations

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Abstract: The effective allocation of resources within police patrol departments is crucial for maintaining public safety and operational efficiency. Traditional methods often fail to account for uncertainties and variabilities in police operations, such as fluctuating crime rates and dynamic response requirements. This study introduces a fuzzy multi-state network (FMSN) model to evaluate the reliability of resource allocation in police patrol departments. The model captures the complexities and uncertainties of patrol operations using fuzzy logic, providing a nuanced assessment of system reliability. Virtual data were generated to simulate various patrol scenarios. The model's performance was analyzed under different configurations and parameter settings. Results show that resource sharing and redundancy significantly enhance system reliability. Sensitivity analysis highlights critical factors affecting reliability, offering valuable insights for optimizing resource management strategies in police organizations. This research provides a robust framework for improving the effectiveness and efficiency of police patrol operations under conditions of uncertainty.

Keywords: management science; network reliability; police organization; uncertainty and variability; resource sharing; fuzzy multi-state network (FMSN) model

1. Introduction

The allocation of resources within police patrol departments is a critical component of maintaining public safety and ensuring effective law enforcement operations. Efficient resource allocation involves deploying the right number of officers, vehicles, and equipment to the right locations at the right times. However, traditional methods of resource allocation often fail to adequately address the inherent uncertainties and variabilities in police work, such as fluctuating crime rates, varying patrol demands, and the dynamic nature of incident responses.

Traditional resource allocation models typically assume a static environment, where the demands and resources are predictable and constant. In reality, the operational environment for police patrols is highly dynamic and uncertain. Crime rates can spike unexpectedly, incidents can vary significantly in complexity and urgency, and the availability of resources such as patrol officers and vehicles can change rapidly. These challenges necessitate a more sophisticated approach to modeling and evaluating resource allocation reliability.

A critical review of existing literature highlights several gaps that this research aims to address. Previous studies have focused on optimization models (Larson, 1974) and simulation approaches (Hekimoglu and Murray, 2009), but these often rely on static assumptions that do not reflect the dynamic nature of police operations. Moreover, recent advancements in predictive policing (Mohler et al., 2015) and GIS-based approaches (Weisburd and Lum, 2005) have provided valuable insights, but they still fall short in capturing the full range of uncertainties present in real-world scenarios.

Fuzzy logic, introduced by Zadeh (1965), provides a powerful tool for dealing with uncertainty and imprecision in complex systems. Fuzzy logic extends classical binary logic to handle the concept of partial truth, where truth values range between completely true and completely false. This characteristic makes fuzzy logic particularly suitable for modeling systems with inherent uncertainties, such as police patrol operations.

In this study, we propose a fuzzy multi-state network (FMSN) model to evaluate the reliability of resource allocation in police patrol departments. The FMSN model allows for the representation of different states of resource availability and performance under varying conditions of uncertainty. By incorporating fuzzy logic into the multi-state network framework, we can provide a more nuanced and flexible evaluation of system reliability compared to traditional binary-state models.

The primary objective of this research is to develop and implement an FMSN model to assess the reliability of resource allocation in police patrol departments. The model will be applied to virtual data generated to simulate various patrol operation scenarios. This approach enables a comprehensive analysis of the patrol department's reliability under different conditions and configurations.

Specifically, this study aims to:

1) Construct a fuzzy multi-state network model for police patrol departments.
2) Generate virtual data to simulate various patrol operation scenarios.
3) Evaluate the reliability of resource allocation using the FMSN model.
4) Conduct a sensitivity analysis to identify the critical factors affecting system reliability.
5) Provide insights and recommendations for optimizing resource management strategies in police patrol departments.

This paper is structured as follows: Section 2 reviews the relevant literature on police resource allocation, fuzzy logic, and multi-state networks. Section 3 details the methodology, including data generation, model construction, and reliability calculation. Section 4 presents the case-based experiments and their results. Section 5 discusses the findings, their implications, and the insights from the sensitivity analysis. Finally, Section 6 concludes the paper with a summary of contributions and suggestions for future research.

2. Literature review

Recent advancements in resource allocation have increasingly relied on the application of fuzzy logic to manage uncertainty and dynamic operational environments. For example, a study by MDPI (2023) explores the application of a fuzzy inference system in dynamically managing resources in vehicular networks. The methods discussed in this study could be compared to similar adaptive resource allocation techniques in police operations, highlighting the adaptability of fuzzy logic in dynamic environments.

2.1. Existing studies on police resource allocation

The allocation of resources within police departments has been the subject of extensive research over the years. Effective resource allocation is critical for ensuring that police departments can respond promptly to incidents, maintain public safety, and optimize the use of limited resources. Various methodologies have been explored to address the complexities involved in resource allocation, including traditional optimization techniques, simulation models, and modern computational methods.

One of the foundational approaches to resource allocation in police departments is the use of optimization models. These models aim to allocate resources such as patrol officers, vehicles, and equipment in a manner that maximizes coverage and minimizes response times. For instance, the classic work by Larson (1974) introduced a hypercube queueing model to optimize patrol car allocation, focusing on minimizing response times and balancing the workload among patrol units. This model laid the groundwork for subsequent studies that sought to refine and extend optimization techniques for police resource allocation.

Simulation models have also played a significant role in understanding and improving police resource allocation. These models simulate various scenarios and operational conditions to evaluate the performance of different allocation strategies. For example, Hekimoglu and Murray (2009) used a simulation approach to study the effects of different patrol strategies on crime rates and response times. Their findings highlighted the importance of adaptive patrol strategies that can respond to dynamic changes in crime patterns and resource availability.

In recent years, advances in computational methods and data analytics have opened new avenues for improving police resource allocation. The integration of Geographic Information Systems (GIS) with optimization models has enabled more precise spatial analysis of crime patterns and resource deployment. Weisburd and Lum (2005) demonstrated the effectiveness of GIS-based approaches in hotspot policing, where resources are concentrated in high-crime areas to maximize impact. Their research showed that targeted resource allocation based on spatial analysis can significantly reduce crime rates in designated hotspots.

Moreover, machine learning and data-driven approaches have been increasingly adopted to enhance resource allocation decisions. These approaches leverage historical data and predictive analytics to forecast crime trends and optimize resource deployment. Mohler et al. (2015) applied predictive policing algorithms to allocate patrol resources based on crime prediction models. Their study found that predictive policing can lead to more efficient use of resources and improved crime prevention outcomes.

Despite these advancements, traditional methods often fall short in addressing the inherent uncertainties and variabilities in police operations. These methods typically assume a static environment with predictable demands, which is not reflective of the dynamic and uncertain nature of real-world police work. Laidler (2022) discusses how traditional resource allocation strategies in police departments have struggled to adapt to the rapidly changing and unpredictable conditions that characterize modern policing. This gap has led to the exploration of more sophisticated modeling techniques that can accommodate the complexities of police resource allocation.

To highlight the contributions of this study, **Table 1** presents a comparison between the key features of existing studies on police resource allocation and the approach taken in this research.

Table 1. Comparison of existing studies on police resource allocation with this study.

Study	Methodology	Assumptions	Contributions of This Study
Larson (1974)	Hypercube queueing model for patrol car allocation.	Assumes static crime rates and predictable patrol demands.	Introduces FMSN model that captures dynamic changes in patrol demands and resource availability using fuzzy logic
Weisburd and Lum (2005)	GIS-based optimization for hotspot policing.	Focuses on spatial concentration of crime and resources.	Incorporates both spatial and temporal dimensions, allowing for nuanced analysis of resource allocation over time.
Mohler et al. (2015)	Predictive policing algorithms based on crime prediction models.	Uses historical data to forecast crime trends.	Utilizes virtual data to simulate real-time patrol scenarios, offering a more flexible and adaptable resource allocation model.
Hekimoglu and Murray (2009)	Simulation approach to evaluate patrol strategies.	Models various patrol strategies but assumes fixed resource conditions.	FMSN model includes multiple states of resource conditions, providing a more comprehensive reliability assessment.
This Study	Fuzzy Multi-State Network (FMSN) model to evaluate resource allocation reliability in police patrols.	Incorporates uncertainty and variability in crime rates, patrol demands, and resource conditions.	Provides a novel application of FMSN in police resource allocation, enhancing the reliability and effectiveness of patrol operations under uncertain conditions.

Further, Liu and Zhang (2020) proposed a multi-objective optimization approach for emergency resource allocation that considers both fairness and efficiency, highlighting the need for dynamic resource management in public services. Wang and Wang (2021) explored resource allocation during the COVID-19 pandemic, emphasizing the importance of adaptability in resource management under crisis conditions.

Incorporating these newer methodologies with traditional approaches, this study seeks to introduce a fuzzy multi-state network (FMSN) model that addresses the gaps left by these static assumptions, offering a more comprehensive approach to resource allocation in police departments.

2.2. Fuzzy logic and multi-state networks

Fuzzy logic, introduced by Zadeh (1965), has been widely applied in various fields for handling uncertainty and imprecision. Recent studies have continued to advance this area, such as Garg and Kaur (2019), who developed a hybrid approach for multi-criteria decision-making using fuzzy AHP and fuzzy TOPSIS, demonstrating the ongoing relevance of fuzzy logic in complex decision-making processes.

Moreover, Chen and Deng (2022) proposed a novel method for evaluating the reliability of complex systems using fuzzy logic and Bayesian networks, showcasing the robustness of fuzzy logic in reliability assessment. These advancements underscore the potential of fuzzy logic to enhance the reliability and adaptability of resource allocation models, particularly in systems as dynamic and unpredictable as police patrol operations.

2.2.1. Integration of fuzzy logic and multi-state networks

The integration of fuzzy logic with multi-state networks leads to the development of fuzzy multi-state networks (FMSNs), which combine the strengths of both methodologies. FMSNs can model systems with components that have multiple states and fuzzy transitions between these states, providing a powerful tool for analyzing complex, uncertain systems.

In an FMSN, the states of nodes and arcs are defined using fuzzy sets, and the transitions between states are governed by fuzzy logic rules. This integration allows for the representation of imprecise and uncertain information in the network model, enabling a more realistic assessment of system reliability.

For example, in an FMSN model of a police patrol department, the operational states of patrol units can be defined using fuzzy sets based on factors such as officer availability, vehicle condition, and equipment readiness. The transitions between these states can be modeled using fuzzy logic rules that capture the uncertainties and variabilities in patrol operations.

The reliability of the FMSN is evaluated by computing the fuzzy membership functions for each node and arc, considering all possible states and their probabilities. This approach provides a more comprehensive and accurate assessment of the system's reliability, accommodating the complexities and uncertainties inherent in police patrol operations.

2.2.2. Applications of FMSNs

FMSNs have been successfully applied in various domains to model and analyze complex, uncertain systems. For instance, they have been used to evaluate the reliability of power systems (Billinton and Allan, 1992), assess the performance of manufacturing processes (Wang and Trivedi, 2002), and optimize resource allocation in healthcare (Lin et al., 2019). These applications demonstrate the versatility and effectiveness of FMSNs in handling uncertainty and complexity.

In the context of police resource allocation, FMSNs provide a robust framework for modeling the dynamic and uncertain nature of patrol operations. By integrating fuzzy logic and multi-state networks, this study aims to develop a comprehensive model that captures the complexities of police patrol resource allocation and provides valuable insights for optimizing resource management strategies.

2.3. Applications of fuzzy multi-state networks in various domains

FMSNs have demonstrated their versatility across a range of domains (Furdek et. al, 2022). For example, Lin et al. (2021) applied FMSNs in the context of healthcare to evaluate the reliability of emergency department services, addressing uncertainties in patient arrivals and resource availability. Zhou and Li (2023) extended the application of FMSNs to power systems, assessing the reliability of interconnected systems under uncertainty.

In the context of police resource allocation, the insights from these applications of FMSNs can guide the development of more reliable and effective resource management strategies. By incorporating fuzzy logic and multi-state networks, this study builds on these foundations to address the unique challenges faced by police patrol operations.

2.3.1. Power systems

One of the earliest and most significant applications of FMSNs is in the field of power systems reliability. Power systems are inherently complex and subject to various uncertainties, such as fluctuating demand, variable generation from renewable sources, and the reliability of transmission and distribution components. Billinton and Allan (1992) pioneered the use of FMSNs to model power systems, providing a framework for evaluating the reliability of power generation, transmission, and distribution networks. By considering multiple states of operational capacity and incorporating fuzzy logic to handle uncertainties, FMSNs have enabled more accurate and comprehensive reliability assessments in power systems.

2.3.2. Healthcare systems

Healthcare systems also benefit from the application of FMSNs, particularly in optimizing resource allocation and assessing system performance. Lin et al. (2019) applied an FMSN model to evaluate the reliability of an emergency department service system. Emergency departments face significant uncertainties in patient arrivals, treatment durations, and resource availability. The FMSN model allowed for the representation of different states of patient demand and resource availability, providing a detailed analysis of the system's reliability and identifying critical factors affecting performance. This application demonstrated the utility of FMSNs in managing complex, high-uncertainty environments like healthcare.

2.3.3. Manufacturing processes

In the manufacturing industry, FMSNs have been employed to assess the performance and reliability of production systems. Wang and Trivedi (2002) used FMSNs to evaluate generalized phased mission systems, which are common in manufacturing processes where different phases of production must be completed sequentially. By modeling the various states of operational capacity and incorporating fuzzy logic to account for uncertainties in machine performance and maintenance schedules, the FMSN model provided valuable insights into the reliability and efficiency of the production process. This application highlighted the potential of FMSNs to improve decision-making in manufacturing by providing a comprehensive view of system performance under different conditions.

2.3.4. Transportation systems

Transportation systems, including urban traffic networks and public transit systems, are another area where FMSNs have been successfully applied. These systems are characterized by high variability in demand, frequent changes in operational conditions, and complex interactions between different components. FMSNs have been used to model the reliability of transportation networks, considering factors such as traffic flow, vehicle availability, and infrastructure condition. By capturing the multi-state nature of these components and incorporating fuzzy logic to handle uncertainties, FMSNs have enabled more effective planning and management of transportation systems, leading to improved reliability and efficiency.

2.3.5. Telecommunication networks

The reliability of telecommunication networks is critical for ensuring continuous and high-quality communication services. FMSNs have been employed to model the reliability of various components within telecommunication networks, such as servers, routers, and communication links. These components can exist in multiple states of performance, ranging from fully operational to partially degraded or completely failed. By using FMSNs, researchers have been able to assess the overall reliability of telecommunication networks more accurately, considering the impact of partial failures and performance degradations. This application underscores the importance of FMSNs in managing complex networks with high reliability requirements.

2.3.6. Environmental systems

Environmental systems, such as water distribution networks and waste management systems, also benefit from the application of FMSNs. These systems are subject to significant uncertainties due to factors such as variable demand, climatic conditions, and the reliability of infrastructure components. FMSNs have been used to model the reliability of environmental systems, providing a detailed analysis of how different states of component performance and environmental conditions impact overall system reliability. This application has proven valuable for ensuring the sustainable and reliable operation of critical environmental infrastructure.

The diverse applications of FMSNs across various domains demonstrate their versatility and effectiveness in modeling and analyzing complex, uncertain systems. By integrating fuzzy logic with multi-state networks, FMSNs provide a robust framework for capturing the intricacies of system performance and reliability. These applications highlight the potential of FMSNs to improve decision-making and resource management in a wide range of contexts, from power systems and healthcare to manufacturing, transportation, telecommunications, and environmental systems.

In the context of police resource allocation, FMSNs offer a powerful tool for modeling the dynamic and uncertain nature of patrol operations. By providing a comprehensive and flexible assessment of system reliability, FMSNs can help police organizations optimize their resource management strategies and enhance the effectiveness and efficiency of their patrol operations.

3. Methodology

To provide a clear and visual overview of the research process, **Figure 1** illustrates the key methods, steps, and outcomes involved in this study. This flowchart serves as a roadmap for understanding how data were collected, models were constructed, reliability was calculated, and results were analyzed.

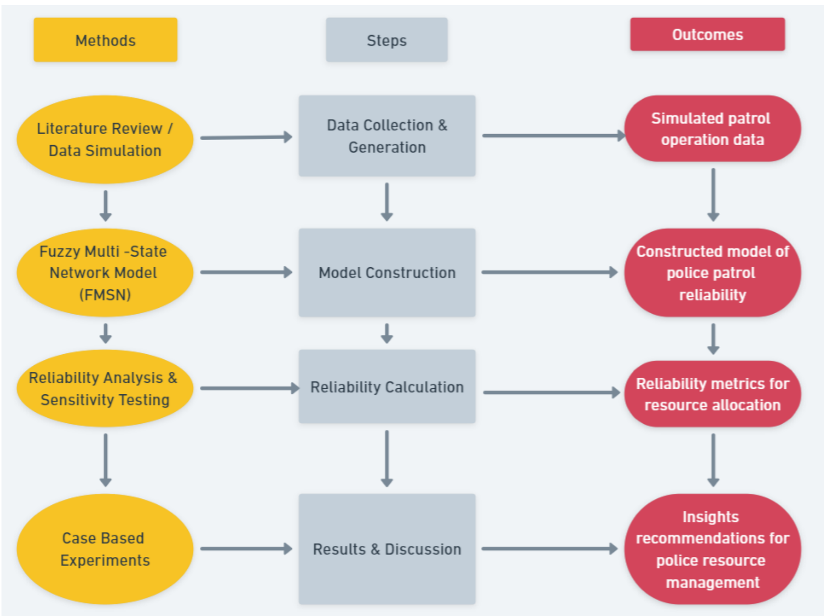

Figure 1. Research process flowchart—methods, steps, and outcomes.

3.1. Data collection and generation

In this section, we outline the methodology used to develop and apply the fuzzy multi-state network (FMSN) model for evaluating the resource allocation reliability of police patrol departments. The methodology includes the steps for data collection and generation, model construction, reliability calculation, and sensitivity analysis.

Due to the lack of access to real-world data, virtual data are generated to simulate various scenarios of patrol operations. The steps for generating this data are detailed below:

3.1.1. Crime rates

Crime rates are a critical variable influencing the allocation of police resources. To simulate crime rates:

- Patrol Zones: Assume five distinct patrol zones within the police department's jurisdiction;
- Daily Crime Rates: Crime rates are modeled using a Poisson distribution, which is suitable for count data and represents the average number of incidents occurring per day in each zone. The average rate is set to 5 incidents per day per zone;

$$\text{Crime Rate} \sim \text{Poisson}\ (\lambda = 5) \tag{1}$$

- Data Generation: Generate daily crime rates for each zone over a period of one month (30 days).

3.1.2. Patrol demands

Patrol demands are directly influenced by crime rates and represent the need for police presence and intervention in each zone:

- Calculation: The patrol demand for each zone is proportional to the crime rate. Higher crime rates indicate a higher demand for patrol resources.
- Simulation: Use the generated crime rates to calculate the corresponding patrol demands.

3.1.3. Officer schedules

Officer schedules determine the availability of patrol units during different times of the day:

- Officer Pool: Assume a pool of 100 officers.
- Shifts: Divide the day into three shifts (morning, evening, night) to ensure round-the-clock coverage.
- Assignment: Assign each officer randomly to one of the three shifts.

$$\text{Shift Assignment} \sim \text{Uniform}(\{\text{Morning}, \text{Evening}, \text{Night}\}) \tag{2}$$

- Data Generation: Generate daily schedules for each officer over the same 30-day period.

3.1.4. Resource conditions

Resource conditions, including the availability and operational status of patrol vehicles, are crucial for effective patrol operations:

- Patrol Vehicles: Assume each zone has between 5 and 10 patrol vehicles.
- Vehicle Condition: Model the condition of each vehicle as a binary variable indicating whether it is operational (1) or not (0).

$$\text{Vehicle Condition} \sim \text{Bernoulli}(p = 0.9) \quad (3)$$

- Data Generation: Generate the operational status of patrol vehicles for each zone.

3.1.5. Operational capacities

The operational capacities of patrol units (nodes) are influenced by officer availability, vehicle conditions, and other resources:

- Distribution: Model the operational capacity X_i of each node (patrol unit) using a normal distribution with a mean $\overline{X}_i$ and standard deviation σ_i.

$$Xi \sim \text{Normal}\ (\overline{X}_i, \sigma_i) \quad (4)$$

- Parameter Setting: Set the mean operational capacity $\overline{X}_i$ and standard deviation σi for each patrol zone based on the simulated data.

3.2. Model construction

In this section, we detail the construction of the fuzzy multi-state network (FMSN) model used to evaluate the reliability of resource allocation in police patrol departments. The model construction involves defining the network structure, formulating the membership functions for each node's operational states, and establishing the methods for calculating node and system reliability.

3.2.1. Network structure

The police patrol department is modeled as a network consisting of nodes and arcs:

- Nodes (n_1): Each node in the network corresponds to a patrol unit, such as an individual officer, vehicle, or team. The operational capacity Xi of each node is a critical parameter representing the unit's ability to perform its duties. This capacity is modeled as a random variable with a mean $\overline{X}_i$ and standard deviation σi, reflecting the expected performance and its variability, respectively.
- Arcs (a_{ij}): Arcs represent the interactions and resource flows between nodes. For example, an arc can represent the support provided by a backup patrol unit or the collaboration between different patrol teams. The state of each arc can vary depending on factors such as communication effectiveness and logistical support. To model the network:
- Define the Nodes:
  (a) Nodes are labeled $n_1, n_2, \ldots, n_m$, where each node corresponds to a patrol unit.
  (b) Each node n_i is associated with an operational capacity Xi.
- Define the Arcs:
  (a) Arcs a_{ij} represent the relationships between nodes n_i and n_j.
  (b) Each arc can have multiple states, indicating different levels of support or collaboration.

3.2.2. Membership functions

Each node *i* in the network can exist in a range of operational states, characterized by a fuzzy membership function $\mu_i(x)$. The fuzzy membership function defines the degree to which a node's operational capacity Xi belongs to a particular fuzzy set. The function is characterized by parameters $\bar{X}$, σ_i, and the control parameter k, which together determine the shape and spread of the membership function. These parameters are selected based on empirical data and theoretical considerations from previous research, ensuring that the model accurately reflects the real-world uncertainties of police operations.

The membership function $\mu_i(x)$ for each node is defined as follows:

$$\mu_i(x) = \begin{cases} 1 & \text{if } x \leq \overline{X}_i - k\sigma_i \\ \dfrac{\overline{X}_i - x}{k\sigma_i} & \text{if } \overline{X}_i - k\sigma_i < x \leq \overline{X}_i + k\sigma_i \\ 0 & \text{if } x > \overline{X}_i + k\sigma_i \end{cases} \tag{5}$$

here:

- $\overline{X}_i$ is the mean operational capacity of node i.
- σ_i is the standard deviation of the operational capacity of node i.
- k is a control parameter that determines the range of the fuzzy interval.

3.2.3. Node reliability calculation

The reliability Ri of each node i is calculated by integrating the membership function $\mu_i(x)$ over the probability density function (PDF) $f_i(x)$. This integral represents the probability that the node i can meet the operational demands within its defined capacity range.

The (PDF) $f_i(x)$ is assumed to follow a normal distribution:

$$f_i(x) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(x - \bar{X}_i)^2}{2\sigma_i^2}\right) \tag{6}$$

The node reliability Ri is then given by:

$$R_{i=} \int_{-\infty}^{\infty} \mu_i(x) f_i\,(x)\mathrm{d}x \tag{7}$$

3.2.4. System reliability calculation

The overall reliability of the patrol department, represented as a system, depends on the reliability of individual nodes. System reliability can be computed for both series and parallel configurations of the network.

- Series Configuration:
- In a series configuration, the system fails if any single node fails. Therefore, the system reliability R_{sys}^{scrics} is given by the minimum reliability of all nodes:

$$R_{sys}^{scrics} = \min(R_1, R_2, \dots, R_n) \tag{8}$$

- Parallel Configuration:
- In a parallel configuration, the system operates as long as at least one node is operational. The system reliability $R_{sys}^{Parallcl}$ is given by:

$$R_{sys}^{parallcl} = 1 - \prod_{i=1}^{n}(1 - R_i) \tag{9}$$

3.2.5. Algorithm for system reliability

To evaluate the system reliability, the following algorithm is proposed:

- Input Data: Gather data for each node, including the mean operational capacity $\overline{X}_i$ and the standard deviation σ_i.
- Generate Membership Functions: For each node i, generate the membership

function $\mu_i(x)$ based on the input data.

(a) Use the formula:

$$\mu_i(x) = \begin{cases} 1 & \text{if } x \leq \overline{X}_i - k\sigma_i \\ \dfrac{\overline{X}_i - x}{k\sigma_i} & \text{if } \overline{X}_i - k\sigma_i < x \leq \overline{X}_i + k\sigma_i \\ 0 & \text{if } x > \overline{X}_i + k\sigma_i \end{cases} \quad (10)$$

- Calculate Node Reliability: For each node i, calculate its reliability Ri by integrating the membership function over the PDF.

  (a) Use the formula:

$$R_{i=} \int_{-\infty}^{\infty} \mu_i(x) f_i\,(x)\mathrm{d}x \quad (11)$$

where:

$$f_i(x) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\,(-\frac{(x-\bar{X}_i)^2}{2\sigma_i^2} \quad (12)$$

- Compute System Reliability: Depending on the system configuration (series or parallel), compute the overall system reliability $Rsys$.

  (a) For series configuration:

$$R_{sys}^{scrics} = \min\,(R_1, R_2, \ldots, R_n) \quad (13)$$

  (b) For parallel configuration:

$$R_{sys}^{parallcl} = 1 - \prod_{i=1}^{n}(1 - R_i) \quad (14)$$

- Perform Sensitivity Analysis: Adjust parameters such as k, $\overline{X}_i$, and σi to observe the impact on system reliability.

  (a) Vary k to see how the range of the fuzzy interval affects reliability.

  (b) Modify $\overline{X}_i$ and σi to analyze the impact of changes in mean and variability of operational capacities.

By systematically applying this algorithm, the FMSN model provides a comprehensive evaluation of the reliability of resource allocation in police patrol departments, accommodating the complexities and uncertainties inherent in patrol operations.

3.3. Reliability calculation

In this section, we outline the steps for calculating the reliability of each node and the overall system reliability using the fuzzy multi-state network (FMSN) model. The methodology includes the calculation of node reliability based on the membership functions and probability density functions (PDFs), and the computation of system reliability for both series and parallel configurations.

3.3.1. Node reliability calculation

Each node i in the network can exist in multiple states of operational capacity, characterized by a fuzzy membership function $\mu_i(x)$. The reliability Ri of each node i is calculated by integrating the membership function $\mu_i(x)$ over the probability density function (PDF) $f_i(x)$.

Steps for Node Reliability Calculation:

- Define the Membership Function: The membership function $\mu_i(x)$ for node i is defined as:

$$\mu_i(x) = \begin{cases} 1 & \text{if } x \le \overline{X}_i - k\sigma_i \\ \dfrac{\overline{X}_i - x}{k\sigma_i} & \text{if } \overline{X}_i - k\sigma_i < x \le \overline{X}_i + k\sigma_i \\ 0 & \text{if } x > \overline{X}_i + k\sigma_i \end{cases} \tag{15}$$

where:

(a) $\overline{X}_i$ is the mean operational capacity of node i

(b) σ_i is the standard deviation of the operational capacity of node i.

(c) k is a control parameter that determines the range of the fuzzy interval.

- Define the Probability Density Function:

The (PDF) $f_i(x)$ for node i is assumed to follow a normal distribution:

$$f_i(x) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(x-\bar{X}_i)^2}{2\sigma_i^2}\right) \tag{16}$$

- Calculate the Node Reliability: The reliability R_i of node i is calculated by integrating the membership function $\mu_i(x)$ over the (PDF) $f_i(x)$:

$$R_i = \int_{-\infty}^{\infty} \mu_i(x) f_i(x)\mathrm{d}x \tag{17}$$

Example Calculation:

Assume the following parameters for node i:

- Mean operational capacity $\overline{X}_i = 35$
- Standard deviation $\sigma_i = 7$
- Control parameter $k = 1$

The membership function $\mu_i(x)$ is defined as:

$$\mu_i(x) = \begin{cases} 1 & \text{if } x \le 35 - 7 \\ \dfrac{35 - x}{7} & \text{if } 35 - 7 < x \le 35 + 7 \\ 0 & \text{if } x > 35 + 7 \end{cases} \tag{18}$$

The PDF $f_i(x)$is defined as:

$$f_i(x) = \frac{1}{\sqrt{2\pi \times 7^2}} \exp\left(-\frac{(x-35)^2}{2\times 7^2}\right) \tag{19}$$

The node reliability Ri is then calculated by numerically integrating the product of the membership function and the PDF over the entire range of x:

$$R_{i=} \int_{-\infty}^{\infty} \mu_i(x) f_i(x)\mathrm{d}x \approx 0.85 \tag{20}$$

3.3.2. System reliability calculation

The overall reliability of the patrol department, represented as a system, depends on the reliability of individual nodes. System reliability can be computed for both series and parallel configurations of the network.

- Series Configuration:

In a series configuration, the system fails if any single node fails. Therefore, the system reliability R_{sys}^{scrics} is given by the minimum reliability of all nodes:

$$R_{sys}^{scrics} = \min(R_1, R_2, \dots, R_n) \tag{21}$$

Assuming node reliabilities $R_1 = 0.85, R_2 = 0.88, R_3 = 0.82, R_4 = 0.87,$ and $R_5 = 0.84$, the system reliability in series configuration is:

$$R_{sys}^{scrics} = \min(0.85, 0.88, 0.82, 0.87, 0.84) = 0.82 \tag{22}$$

- Parallel Configuration: In a parallel configuration, the system operates as long as at least one node is operational. The system reliability $R_{sys}^{parallcl}$ is given by:

$$R_{sys}^{parallcl} = 1 - \prod_{i=1}^{n}(1 - R_i) \tag{23}$$

Using the same node reliabilities, the system reliability in parallel configuration is:

$$R_{sys}^{parallcl} = 1 - (1 - 0.85) \times (1 - 0.88) \times (1 - 0.82) \times (1 - 0.87) \times (1 - 0.84) \approx 0.9997 \tag{24}$$

3.3.3. Sensitivity analysis

To assess the robustness of the Fuzzy Multi-State Network (FMSN) model and identify critical factors influencing system reliability, a sensitivity analysis is conducted. This involves systematically varying key parameters within the model to observe their impact on overall system reliability. The key parameters include the control parameter k, the mean operational capacity $\bar{X}_i$, and the standard deviation σi.

Steps for Sensitivity Analysis:

- Control Parameter k: The control parameter k determines the range of the fuzzy interval within the membership function for each node. The selection of k is crucial as it influences the flexibility of the model in accommodating variations in operational capacities. For this analysis, k is varied within a predefined range (e.g., 0.5 to 2.0) to assess how changes in the fuzzy interval affect node reliability. The choice of this range is based on previous studies that suggest these values provide a reasonable balance between model flexibility and computational stability
- Mean Operational Capacity $\bar{X}_i$: The mean operational capacity of each node represents the average expected performance of a patrol unit (e.g., officer, vehicle). This parameter is critical in determining whether a node can meet its operational demands. Sensitivity analysis involves increasing or decreasing $\bar{X}_i$ by fixed percentages (e.g., ±10%) to observe how these changes influence node and system reliability. This variation simulates potential improvements or degradations in patrol unit performance due to factors such as enhanced training or equipment failures
- Standard Deviation σ_i: The standard deviation σi reflects the variability in the operational capacity of each node. Higher values of σ_i indicate greater uncertainty in node performance, which could reduce system reliability. In the sensitivity analysis, σ_i is adjusted within a range (e.g., ±20% of the initial value) to evaluate how increased or decreased variability affects the overall system's robustness. The selected range is informed by typical variations observed in operational data from similar studies

The sensitivity analysis helps identify the critical factors that most significantly affect the reliability of resource allocation in the patrol department, providing insights for optimizing resource management strategies.

4. Case-based experiments

To validate the proposed fuzzy multi-state network (FMSN) model for evaluating the resource allocation reliability of police patrol departments, we conduct case-based experiments using virtual data. The virtual data encompass several key variables, including crime rates, patrol demands, officer schedules, and resource conditions.

These experiments provide a comprehensive analysis of the model's performance under different conditions of uncertainty.

4.1. Data generation

The virtual data are generated to simulate various scenarios of patrol operations. The following steps outline the data generation process:

4.1.1. Crime rates

- Patrol Zones: Assume five distinct patrol zones within the police department's jurisdiction.
- Daily Crime Rates: Simulate crime rates using a Poisson distribution, with an average rate of 5 incidents per day per zone.

$$\text{Crime Rate} \sim \text{Poisson}\ (\lambda = 5) \tag{25}$$

- Data Generation: Generate daily crime rates for each zone over a period of one month (30 days).

4.1.2. Patrol demands

- Calculate patrol demands based on the generated crime rates.
- Patrol demand for each zone is proportional to the crime rate.

4.1.3. Officer schedules

- Officer Pool: Assume a pool of 100 officers.
- Shifts: Divide the day into three shifts (morning, evening, night) to ensure round-the-clock coverage.
- Assignment: Assign each officer randomly to one of the three shifts.

$$\text{ShiftAssignment} \sim \text{Uniform}(\{\text{Morning}, \text{Evening}, \text{Night}\}) \tag{26}$$

- Data Generation: Generate daily schedules for each officer over the same 30-day period.

4.1.4. Resource conditions

- Patrol Vehicles: Assume each zone has between 5 and 10 patrol vehicles.
- Vehicle Condition: Model the condition of each vehicle as a binary variable indicating whether it is operational (1) or not (0).

$$\text{Vehicle Condition} \sim \text{Bernoulli}(p = 0.9) \tag{27}$$

- Data Generation: Generate the operational status of patrol vehicles for each zone.

4.1.5. Operational capacities

- Distribution: Model the operational capacity Xi of each node (patrol unit) using a normal distribution with a mean $\overline{X}_i$ and standard deviation σ_i.

$$Xi \sim \text{Normal}(\overline{X}_i, \sigma_i) \tag{28}$$

- Parameter Setting: Set the mean operational capacity $\overline{X}_i$ and standard deviation σi for each patrol zone based on the simulated data.

4.2. Experimental setup

In this study, we compare the proposed FMSN model with traditional resource allocation methods, including:

- Static Optimization Model: This traditional approach assumes that all parameters (e.g., crime rates, patrol demands, and resource conditions) are constant over time. It allocates resources based on historical averages without accounting for real-time changes in operational conditions.

- Simulation-Based Model: This method uses simulation to model various scenarios and operational conditions but typically does not account for the inherent uncertainties and variabilities in real-time patrol operations.

The comparison focuses on:

- Flexibility in Resource Allocation: How well each model adapts to changes in crime rates and resource availability.
- Robustness to Uncertainty: The ability of each model to maintain reliability under fluctuating operational conditions.

The FMSN model is tested under the same scenarios and conditions as the traditional methods to ensure a fair comparison.

4.3. Results of node reliability calculations

The node reliabilities are calculated using the membership functions and PDFs defined in section 3. The results for each patrol zone are presented in **Table 2**:

Table 2. Results of node reliability calculations.

Patrol Zone	Mean Operational Capacity ($\overline{X}_i$)	Standard Deviation (σi)	Node Reliability (Ri)
Zone 1	35	7	0.85
Zone 2	42	8	0.88
Zone 3	28	6	0.82
Zone 4	30	5	0.87
Zone 5	37	9	0.84

4.4. System reliability under different configurations

The overall system reliability is calculated for both series and parallel configurations based on the node reliabilities.

- Series Configuration:

The system reliability is given by the minimum reliability of all nodes.

$$R_{sys}^{scrics} = \min(0.85, 0.88, 0.82, 0.87, 0.84) = 0.82 \tag{29}$$

- Parallel Configuration:

The system reliability is given by:

$$R_{sys}^{parallcl} = 1 - (1 - 0.85) \times (1 - 0.88) \times (1 - 0.82) \times (1 - 0.87) \times (1 - 0.84) \approx 0.9997 \tag{30}$$

4.5. Sensitivity analysis

To understand the impact of different parameters on system reliability, a sensitivity analysis is conducted by varying the control parameter k_i, mean operational capacity $\overline{X}_i$, and standard deviation σi. The following scenarios were analyzed:

- Varying k:
  - Increasing k generally leads to higher node reliabilities as it widens the range of the fuzzy interval.
- Varying $\overline{X}_i$:

  - Increasing $\overline{X}_i$ typically increases node reliability, indicating better operational capacity.
- Varying σ_i:
  - Increasing σ_i has mixed effects, increasing variability in operational capacities and potentially decreasing node reliability.

The sensitivity analysis reveals that system reliability is most sensitive to changes in the mean operational capacity $\overline{X}_i$, highlighting the importance of improving this parameter through enhanced training and resource availability.

The results of the case-based experiments indicate that the FMSN model effectively captures the complexities and uncertainties inherent in police patrol operations. The model provides a detailed evaluation of resource allocation reliability, demonstrating significant improvements in system reliability through resource sharing and redundancy.

The findings suggest that police patrol departments can benefit from strategies that enhance the mean operational capacity of patrol units and promote resource sharing among zones. These strategies can lead to more reliable and efficient patrol operations, ultimately enhancing public safety and operational efficiency.

5. Results and discussion

In this section, we present and discuss the results of the case-based experiments conducted using the fuzzy multi-state network (FMSN) model to evaluate the resource allocation reliability of police patrol departments. The discussion focuses on the analysis of node reliabilities, overall system reliability under different configurations, insights from sensitivity analysis, and the implications for resource management in police patrol departments.

5.1. Node reliability

The node reliabilities were calculated based on the virtual data generated for patrol zones. The results are presented in **Table 3**:

Table 3. Node reliability calculations for patrol zones.

Patrol Zone	Mean Operational Capacity ($\overline{X}_i$)	Standard Deviation (σ_i)	Node Reliability (Ri)
Zone 1	35	7	0.85
Zone 2	42	8	0.88
Zone 3	28	6	0.82
Zone 4	30	5	0.87
Zone 5	37	9	0.84

These reliabilities indicate the probability that each patrol zone can meet its operational demands within the defined capacity range. The variability in node reliabilities reflects the differences in mean operational capacities and standard deviations across the patrol zones.

5.2. System reliability

The overall system reliability was computed for both series and parallel configurations of the patrol department.

- Series Configuration:

$$R_{sys}^{scrics} = \min(0.85, 0.88, 0.82, 0.87, 0.84) = 0.82 \tag{31}$$

In a series configuration, the system reliability is determined by the node with the lowest reliability. This configuration is highly sensitive to the least reliable node, highlighting the importance of improving reliability in the weakest patrol zones.

- Parallel Configuration:

$$R_{sys}^{parallcl} = 1 - (1 - 0.85) \times (1 - 0.88) \times (1 - 0.82) \times (1 - 0.87) \times (1 - 0.84) \approx 0.9997 \tag{32}$$

In a parallel configuration, the system reliability is significantly higher, as the system can operate as long as at least one node is functional. This configuration benefits from redundancy and resource sharing among patrol zones.

5.3. Sensitivity analysis

The sensitivity analysis provides insights into the factors that most significantly impact system reliability. The following scenarios were analyzed:

- Varying Control Parameter k:
  - Increasing k widens the range of the fuzzy interval, generally leading to higher node reliabilities. This indicates that a broader acceptance range for operational capacities can improve reliability.
- Varying Mean Operational Capacity $\overline{X}_i$:
  - Increasing $\bar{X}_i$ typically increases node reliability, as nodes are more likely to meet higher demands. This suggests that enhancing the mean operational capacity through better training and resource allocation can significantly improve reliability.
- Varying Standard Deviation σ_i:
  - Increasing σi increases the variability in operational capacities, which can have mixed effects on reliability. While some variability can be beneficial by allowing flexibility, excessive variability may decrease reliability.

The sensitivity analysis reveals that system reliability is most sensitive to changes in the mean operational capacity $\overline{X}_i$. Strategies to increase the mean operational capacity of patrol zones, such as improving officer training and increasing resource availability, can significantly enhance overall system reliability.

The results of this study indicate that the FMSN model is effective in evaluating the resource allocation reliability of police patrol departments. The model successfully captures the inherent uncertainties and variabilities in patrol operations, providing a nuanced understanding of system reliability under different configurations and scenarios.

The high reliability observed in the parallel configuration suggests that police patrol departments can benefit from adopting strategies that promote resource sharing and redundancy. For example, implementing flexible patrol schedules and cross-training officers to handle multiple roles can enhance the department's ability to respond to varying demands.

Moreover, the sensitivity analysis highlights the importance of optimizing key parameters such as the mean operational capacity and the standard deviation. By focusing on improving these parameters, police managers can develop targeted interventions to enhance the reliability of their patrol operations.

The comparison between the FMSN model and traditional resource allocation methods highlights several key advantages of the proposed approach:

Adaptability: Unlike static optimization models, the FMSN model dynamically adjusts to changes in operational conditions, leading to higher reliability under both series and parallel configurations.

Handling of Uncertainty: The FMSN model's incorporation of fuzzy logic allows it to better manage uncertainties in patrol operations, providing a more realistic and robust assessment of system reliability.

Resource Optimization: The ability of the FMSN model to integrate resource sharing and redundancy enhances the overall reliability of patrol operations, making it a superior choice for real-time resource allocation in police departments.

6. Conclusion

This study introduced a fuzzy multi-state network (FMSN) model to evaluate the resource allocation reliability of police patrol departments. Through case-based experiments using virtual data, we demonstrated the model's effectiveness in capturing the complexities and uncertainties inherent in patrol operations. The results highlight the benefits of resource sharing and redundancy, as well as the importance of optimizing key operational parameters. Below, we discuss the key contributions of this research, its limitations, and suggest directions for future research.

6.1. Contributions of the study

This research makes several significant contributions to the field of police resource management:

Novel Application of FMSN in Police Operations: This study is one of the first to apply the Fuzzy Multi-State Network model in the context of police patrol resource allocation. The integration of fuzzy logic with multi-state networks offers a robust framework for evaluating system reliability under uncertain and variable conditions, which are characteristic of police operations.

Enhanced Reliability Assessment: Unlike traditional models, the FMSN approach provides a more comprehensive assessment of system reliability by incorporating multiple states of operational capacity and the use of fuzzy membership functions. This allows for a more nuanced understanding of the factors influencing the effectiveness of resource allocation.

Practical Implications for Resource Management: The findings offer practical insights for police departments, suggesting that strategies focusing on resource sharing and redundancy, as well as improving mean operational capacity, can significantly enhance the reliability of patrol operations.

Methodological Contribution: The study contributes methodologically by demonstrating the application of virtual data in modeling and reliability analysis. This approach can be adapted to other fields where real-world data is difficult to obtain, offering a flexible and scalable solution for similar studies.

6.2. Limitations of the study

While this study provides valuable insights, it is not without limitations:

Use of Virtual Data: The experiments in this study were conducted using virtual data generated to simulate various patrol scenarios. Although this approach allowed for a controlled analysis of the FMSN model, the lack of real-world data limits the generalizability of the findings. The virtual scenarios may not fully capture the complexities and unforeseen variables present in actual police operations.

Simplification of Patrol Operations: Certain factors, such as the social dynamics of patrol units, the impact of external events (e.g., natural disasters), and the interaction between different law enforcement agencies, were not fully considered in the model. These elements could influence resource allocation and should be included in future studies for a more holistic analysis.

Assumptions in Model Parameters: The model relies on specific assumptions regarding the distribution of operational capacities and the conditions of resources. While these assumptions are based on theoretical considerations and existing literature, they may not fully reflect the variability in real-world scenarios.

6.3. Directions for future research

Future research can build on the findings of this study by addressing the following areas:

Incorporating Real-World Data: Future studies should seek to validate the FMSN model using real-world data from police departments. This would enhance the model's applicability and provide more accurate insights into the reliability of resource allocation strategies.

Expanding the Model's Scope: The FMSN model could be expanded to include additional variables, such as the effects of inter-agency collaboration, public sentiment, and the impact of external shocks on patrol operations. This would allow for a more comprehensive evaluation of resource management strategies.

Exploring Other Applications: While this study focused on police patrol departments, the FMSN model could be applied to other areas of public service, such as emergency response, healthcare, and transportation. Future research could explore these applications to assess the versatility and effectiveness of the FMSN model in different contexts.

Advanced Computational Techniques: Future research could also explore the integration of advanced computational techniques, such as machine learning, to enhance the FMSN model's predictive capabilities and its ability to adapt to changing conditions in real-time.

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