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Article

Limiting capabilities and conceptual design of heat transfer systems

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Abstract: This study estimates minimal entropy production, the corresponding distribution of heat exchange surfaces and contact temperatures for heat exchange systems with given total heat load and heat transfer coefficient. The analysis proves that an optimal heat exchange system requires constancy of the temperature ratio for the contact flows and equal heat flows' output temperatures. The requirement that the entropy production in the system be greater than the minimum entropy generation marks the boundary of the reachable set of heat exchange systems.

Keywords: heat exchanger; minimum dissipation; optimal organization; conceptual design; multi-flow systems

1. Introduction

The limiting capabilities of various industrial processes (heat and refrigerating engines, separation systems, chemical reactors, etc.) are usually calculated based on thermodynamic correlations of reversible processes (Carnot's efficiency, reversible work of separation). These estimates are very important but are generally too high, as they don't consider the intensity, contact surfaces and some other factors related to a given productivity and finiteness of dimensions of equipment. In some cases, reversible estimations lose any sense, as in the case of stationary non-equilibrium systems with several reservoirs or systems with matter and energy inflows. So, thermodynamic efficiency estimations of a heat exchanger must consider constraints on the contact surface (total efficiency of heat exchange) and the heat load, which is the amount of heat transmitted from hot to cold flows per unit of time. For the efficiency estimation of such systems, we use the exergic method (see [1,2] etc.), that is rating the system's exergy loss. This exergy loss is proportional to the entropy generation and to the ambient temperature T_0. Exergy loss reaches its minimum at the given heat load and at some temperatures of hot flows at the heat exchanger input. This loss corresponds to the average temperature maximum of the cold flows at the heat exchanger output.

In recent years, methods of finite-time thermodynamics (see [3]-[19]) have been used for a wide variety of thermodynamic processes. They are particularly effective for heat transfer processes for which reversible efficiency estimates are meaningless.

This study estimates the minimum entropy generation (EGM), which is the exergy loss, in a heat exchange system. Such estimation permits:

- determine the influence over the system of such factors as temperature and heat capacity of the flows, heat load, total efficiency of heat exchange, etc.;

- evaluate thermodynamic efficiency for a designed heat exchange system by means of comparison of its entropy production with the minimum given the same constraints;
- use optimal heat exchange conditions in order to approximate a designed system's configuration to the ideal when projecting new systems.

This study's summary is as follows. In **Section 2**, we estimate thermodynamic efficiency for a double-flow heat exchanger [20,21] that is for a heat exchange unit. In **Section 3**, we calculate the EGM for a conjunction of heat exchange units with a limited contact area and a heat load that is for a multiflow heat exchanger. Finally, in **Section 4**, we display several examples of this study's results application.

2. Double-flow heat exchange

Entropy generation in a thermodynamic system can be calculated in two ways. In the first place, for a functioning system, the entropy generation can be obtained by estimating the incoming and outgoing flows' values. In the second place, when designing a new system, its entropy generation can be calculated as a product of the flows and driving forces, by using kinetic relationships, heat and mass transfer coefficients, etc. Initially, we apply the first method and consider the influence of incoming and outgoing flows' values on entropy production in a double-flow heat exchanger.

According to ([2]), differential molar entropy depends on the heat capacity and increases with temperature and pressure as follows:

$$ds = \frac{c_p}{T}dT - \left(\frac{\partial v}{\partial T}\right)_p dp. \tag{1}$$

where c_p is the molar heat capacity for constant pressure, and v is the molar volume. Integrating this equation from the initial to some finite values of temperature and pressure yields the molar entropy increase. If the molar flow rate is known, multiplying it by the molar entropy increase yields the entropy generation, which depends on the flow values. Summing up the values for all the flows gives the entropy production for the whole process.

So, for an ideal gas, where the heat capacity depends only on the temperature and $\left(\partial v / \partial T\right)_p = R / p$, the molar entropy increase is

$$s - s_0 = \int_{T_0}^{T} \frac{c_p}{T}dT - R\ln\frac{p}{p_0}, \tag{2}$$

where R is the universal gas constant.

For liquids with a constant heat capacity and constant pressure

$$s - s_0 = c_p \ln\left(\frac{T}{T_0}\right), \tag{3}$$

Here, the entropy increases σ_i, due to a change in the i-flow's state is equal to the product of its heat capacity times the ratio's logarithm of the absolute temperatures at the system's in- and output.

$$\sigma_i = W_i \ln\left(\frac{T}{T_0}\right), \quad i = 1,2. \tag{4}$$

The entropy production is equal to the total difference of the entropy flows at the system's in- and output $\sigma = \sum \sigma_i$.

So, let us derive the entropy generation for a double-flow heat exchanger with the flows' heat capacities W_1 и W_2, the input temperatures T_{10}, T_{20}, the output temperatures $\overline{T}_1, \overline{T}_2$, and fixed heat load $\overline{q}$:

$$\sigma = \sigma_1 + \sigma_2 = W_1 \ln\left(\frac{T_{10} - \overline{q} / W_1}{T_{10}}\right) + W_2 \ln\left(\frac{\overline{T}_2}{\overline{T}_2 - \overline{q} / W_2}\right). \tag{5}$$

Suppose the first, the hot flow's values, and consequently the entropy production σ_1, are fixed. Then, from (5), we determine the relation between the output heated flow's temperature and the entropy production σ

$$\overline{T}_2 = \frac{\overline{q}}{W_2\left(1 - exp\left[-\frac{\sigma - \sigma_1}{W_2}\right]\right)}. \tag{6}$$

The heated flow's output temperature monotonically increases while the entropy production decreases. Analogical calculation for multiflow heat exchangers determines a similar relation between the entropy production and the heated flows' average temperature based on the flows' heat capacity.

Let us find the minimal entropy production σ for a double-flow heat exchanger, where T_0 is the heating flow's fixed input temperature, W is its heat capacity, $\overline{q}$ is the heat load, and $\overline{\alpha}$ is the integral heat exchange efficiency. Denote l as the current contact reference for the hot flow's element, this contact reference increases from zero to L ; $q(u,T)$ is the heat value transmitted from the heating flow to the heated one at the section l, the heated flow's temperature is u.

The entropy generation takes the following form:

$$\sigma = \int_0^L q(u,T)\left(\frac{1}{u} - \frac{1}{T}\right) dl \to \min_{u(l)} \tag{7}$$

Subject to the constraints

$$\frac{dT}{dl} = -\frac{q(u,T)}{W}, \quad T(0) = T_0, \tag{8}$$

$$\int_0^L q(u,T)dl = \overline{q}. \tag{9}$$

We subject the transformation law $u(l)$ and the heat removal law $q(u,T)$ to the optimal selection in order to verify the possibility of their realization.

As the right-hand side of the Equation (8) conserves the load, substitute

$$dl = -\frac{dTW}{q(u,T)}. \tag{10}$$

Hence

$$\sigma = W\int_{T(L)}^{T_0}\left(\frac{1}{u}-\frac{1}{T}\right)dT \to \min_{u(T)}, \tag{11}$$

$$W\int_{T(L)}^{T_0} dT = \overline{q}, \tag{12}$$

$$W\int_{T(L)}^{T_0} \frac{\overline{dT}}{q(u,T)} = L. \tag{13}$$

Subject to the constraint (12)

$$T(L) = T_0 - \frac{\overline{q}}{W}. \tag{14}$$

If heat capacity W (flow's heat capacity) depends on T, function $W(T)$ must be integrated in (12)–(14). To simplify, consider the heat capacity constant.

Let us write the Lagrange function and the optimality conditions for (11), (13), assuming that the solution is nonsingular

$$L = \left(\frac{1}{u}-\frac{1}{T}\right)+\frac{\lambda}{q},$$
$$\frac{\partial L}{\partial u} = 0 \to -\frac{1}{u^2}-\frac{\lambda}{q^2}\frac{\partial q}{\partial u} = 0 \tag{15}$$

or

$$\left(\frac{q(u,T)}{u}\right)^2 : \frac{\partial q}{\partial u} = -\lambda. \tag{16}$$

The Equations (16) and (13) permit us to find $q^*(T)$ and λ. Let us define them for the Newton heat exchange.

$$q = \alpha(T-u). \tag{17}$$

Hence

$$\alpha\left(\frac{T}{u}-1\right)^2=\lambda,\quad \forall l. \tag{18}$$

The constraint (13) takes the form

$$\int_{T(L)}^{T_0}\frac{dT}{\alpha(T-u)}=\frac{L}{W}. \tag{19}$$

The constraints (18), (19) define $u^*(T,\alpha,\lambda)$ and Lagrange multiplier λ. If the heat exchange efficiency is constant, let us introduce its total value $\overline{\alpha}=\alpha L.$

According to (18) proportion $\frac{u}{T}$ is constant. Hence

$$\frac{u}{T}=m<1. \tag{20}$$

So that (19) takes the following form

$$\int_{T(L)}^{T_0}\frac{dT}{T(1-m)}=\frac{\overline{\alpha}}{W}.$$

Hence

$$m=1-\frac{W}{\overline{\alpha}}\ln\frac{T_0}{T_0-\overline{q}/W}. \tag{21}$$

Equation (8) takes the form

$$\frac{dT}{dl}=-\overline{\alpha}T(1-m)\rightarrow T^*(l)=T_0e^{-\frac{\overline{\alpha}(1-m)}{LW}l},\quad u^*(l)=mT^*(l). \tag{22}$$

The EGM with regard to (21) is

$$\sigma^*=W\left(\frac{1}{m}-1\right)\int_{T(L)}^{T_0}\frac{dT}{T}=\frac{W^2\ln^2\dfrac{T_0}{T_0-\overline{q}/W}}{\overline{\alpha}-W\ln\dfrac{T_0}{T_0-\overline{q}/W}}=\overline{\alpha}\frac{(1-m)^2}{m}. \tag{23}$$

Equations (21) and (23) don't contain the heated flow's rates since the flow's temperature $u^*(l)$ depends on $T^*(l)$ optimality condition (18) and its consequence (22).

For fixed W and T_0, constraint $\sigma\geq\sigma^*$ determines an attainable area for heat exchange processes in the space with coordinates $\sigma,\overline{q},\overline{\alpha}$. This area is situated over the limit for optimal organization of the process (see **Figure 1**). For fixed irreversibility, the area limit corresponds to the heat load maximum and to the heat exchange efficiency minimum.

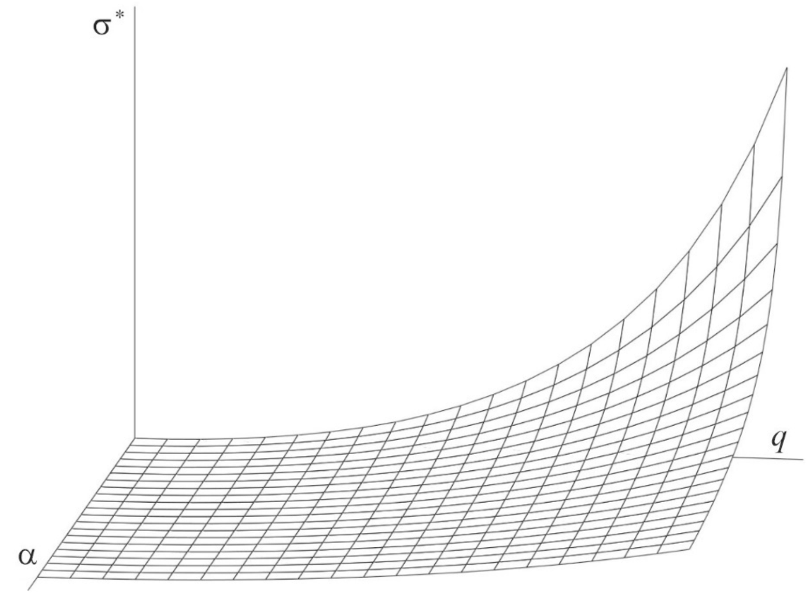

Figure 1. The attainability limit for a heat exchanger with $W = 1, T_0 = 370K$.

It's easily shown (see [21]) that the law of temperature change of the heated flow (22), and consequently the minimum entropy generation (23), can be achieved in a countercurrent tube heat exchanger with a constant lengthwise exchange efficiency α , if the heated flow's heat capacity is

$$W_1 = \frac{W}{m}, \tag{24}$$

and the flow's input temperature is defined (see (5)) as

$$u(L) = T(L)m = \left(T_0 - \frac{\bar{q}}{W}\right)m. \tag{25}$$

Since the entropy production for a random real double-flow heat exchanger is defined as

$$\sigma = W\ln\frac{T_{0\text{out}}}{T_{0\text{in}}} + W_1\ln\frac{T_{1\text{out}}}{T_{1\text{in}}}, \quad \bar{q} = W(T_{0\text{in}} - T_{0\text{out}}), \tag{26}$$

Then, Equation (23) permits to compare σ with σ^*. Where (23) $\bar{\alpha}$ is total heat transfer coefficient for the heat exchanger under consideration; proportion $\eta = \frac{\sigma^*}{\sigma} \le 1$ estimates the heat exchange thermodynamic efficiency.

Example. Find the thermodynamic efficiency for a heat exchanger where hydrodynamics for each flow is characterized as ideally mixed, the heating flow's input temperature is $T_0 = 350K$, its heat capacity is $W = 10$ W/K, the heat exchange efficiency is $\bar{\alpha} = 40$ W/K and the heat load is $\bar{q} = 1000$ W. According to (23), for these conditions, the EGM σ^* equals to 0.31W/K. Taking (26) into account, we have

$$\sigma = W \ln \frac{T_0 - \bar{q}/W}{T_0} + W_1 \ln \frac{T_0 - \bar{q}/W - \bar{q}/\bar{\alpha}}{T_0 - \bar{q}/W - \bar{q}/\bar{\alpha} - \bar{q}/W_1}. \quad (27)$$

For the hot flow's input temperature to be nonnegative, it is required that:

$$W_1 > \frac{\bar{q}}{T_0 - \bar{q}/\alpha - \bar{q}/W} = 4,44 / K.$$

The first component of the right-hand side of this equation is fixed and equals -3.36 W/K. After applying L'Hôpital's rule, we can show that the second component of the right-hand side of Equation (27), when decreasing tends to 4.44W/K, while the heat capacity W_1 tends to infinity. Therefore, efficiency ρ for the heat exchanger under consideration doesn't surpass 0.31/1.08 = 0.29. Relationship $\rho(W_1)$ is shown in **Figure 2**.

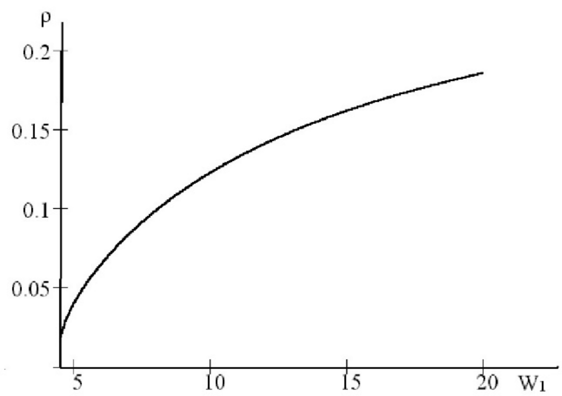

Figure 2. Thermodynamic efficiency dependence on the heat capacity W_1 for a mixing heat exchanger.

3. Multiflow heat exchange

Similar calculations for heat exchange systems with several cooled and heated flows permit the determination of contact flows' temperatures, heat exchange surface distribution and heat loads. Here we use numerous heuristic algorithms [22–27] and others.

Let us obtain the estimation for the EGM in a multiflow heat exchange system, the appropriate temperature change laws for the contacting flows, the heat exchange efficiency distribution and the heat load distribution for the heat exchangers. This estimation permits to find thermodynamic efficiency η for a real working system. On the stage of conceptual design, our calculation allows estimate the future system's characteristics.

Formulation of the problem. Consider the heating flow's input temperature T_0 and heat capacity $W(T_0)$ fixed. Suppose that the flows with temperature T_0 are united in one flow with the total heat capacity

$$W(T_0) = \sum_i g_i c_i,$$

where $g_i(T_0)$ is i-flow's consumption and $c_i(T_0)$ is the flow's heat capacity, with temperature T_0.

Consider function $W(T_0)$ continuous. If the input temperature set is discrete, the calculations will change; we estimate these changes at the section's bottom.

Denote the heating flows' temperature as T_0, then T_{01} is its minimum and T_{02} is its maximum; $q(T_0)$ is the heat load for the flow with temperature T_0; denote the heat transfer efficiency as $\alpha(T_0)$.

The contact surface distribution for the flows is equal to the heat efficiency distribution therefore, consider it fixed.

$$\overline{\alpha} = \int_{T_{01}}^{T_{02}} \alpha(T_0) dT_0, \quad \alpha(T_0) \geq 0, \tag{28}$$

Total heat load is fixed as well

$$\overline{q} = \int_{T_{01}}^{T_{02}} q(T_0) dT_0. \tag{29}$$

If T_0, $W(T_0)$ and $\overline{q}$ are set, then the average enthalpy for the hot flows at the system output is fixed.

The heating flows' output temperatures depend on the input temperature and the heat load as follows

$$T_{\text{out}}(T_0) = T_0 - q(T_0) / W(T_0). \tag{30}$$

Let us define the minimal entropy production

$$\overline{\sigma} = \int_{T_{01}}^{T_{02}} \sigma(T_0) dT_0 \to \min_{u(T,T_0),\alpha(T_0),q(T_0)}, \tag{31}$$

where $u(T, T_0)$ is the temperature of the cold flow when exposed to the hot one with the input temperature T_0 and the current temperature T.

Let us solve problem (28)–(31) in two stages. At the first stage, consider $q(T_0)$ and $\alpha(T_0)$ fixed for all $T_0 \in [T_{01}, T_{02}]$. Hence, define the current temperature correlation for the heated flows u and the heating flows T corresponding to the minimal entropy production $\sigma(T_0)$ for the heating flow with initial temperature T_0. On the second stage, we define the contact surface distribution $\alpha(T_0)$ and the heat load distribution $q(T_0)$, both capable of minimizing $\overline{\sigma}$ on constraints (28) and (29).

The first problem is solved in **Section 2** and results in (21), (23) for every hot flow's input temperature:

$$\frac{u(T,T_0)}{T(T_0)} = m(T_0) = 1 - \frac{W(T_0)}{\alpha(T_0)} \ln \frac{T_0}{T_0 - \frac{q(T_0)}{W(T_0)}}, \tag{32}$$

$$\sigma^*(T_0) = \alpha(T_0) \frac{(1-m(T_0))^2}{m(T_0)}. \tag{33}$$

The second stage results in α and q distribution problem subject to the constraints (28), (29) and the condition

$$\bar{\sigma} = \int_{T_{01}}^{T_{02}} \sigma^*[T_0, \alpha(T_0), W(T_0), q(T_0)]dT_0 \to \min_{\alpha \ge 0,\, q \ge 0} \tag{34}$$

Here, the Lagrange function takes the following form

$$L = \sigma^*(T_0, \alpha, W, q) - \lambda_1 \alpha(T_0) - \lambda_2 q(T_0).$$

where λ_1 and λ_2 are some constants independent of T_0.

The stationarity conditions for L on α and q lead to equations

$$\frac{\partial \sigma^*}{\partial \alpha} = \lambda_1, \quad \frac{\partial \sigma^*}{\partial q} = \lambda_2. \tag{35}$$

In order to calculate derivatives in (35), let us extract the following derivatives

$$\frac{\partial m}{\partial \alpha} = \frac{W(T_0)}{\alpha^2(T_0)} ln \frac{T_0}{T_{\text{out}}} = \frac{1-m(T_0)}{\alpha(T_0)},$$

$$\frac{\partial m}{\partial q} = -\frac{1}{\alpha(T_0) T_{\text{out}}(T_0)},$$

$$\frac{\partial \sigma^*}{\partial m} = \alpha(T_0) \frac{m^2 - 1}{m^2}.$$

Hence (35) takes the following form

$$\frac{\partial \sigma^*}{\partial \alpha} = -\left(\frac{1-m(T_0)}{m(T_0)}\right)^2 = \lambda_1, \tag{36}$$

$$\frac{\partial \sigma^*}{\partial q} = -\frac{m^2(T_0) - 1}{m^2(T_0) T_{\text{out}}(T_0)} = \lambda_2, \tag{37}$$

or

$$T_{\text{out}}(T_0) = \frac{1 - m^2(T_0)}{m^2(T_0)\lambda_2}. \tag{38}$$

From (36) it follows that under condition of optimal heat exchange organization m is independent of T_0 and as it follows from (38) is equal for all the flows. Here, the output temperature is $T_{\text{out}}(T_0) = \overline{T}$.

$\overline{T}$ is unambiguously determined by (29), since

$$\overline{q} = \int_{T_{01}}^{T_{02}} W(T_0)(T_0 - \overline{T})dT_0. \tag{39}$$

Define

$$\overline{W} = \int_{T_{01}}^{T_{02}} W(T_0)dT_0, \tag{40}$$

$$\overline{T_0 W} = \int_{T_{01}}^{T_{02}} T_0 W(T_0)dT_0, \tag{41}$$

then

$$\overline{T} = \frac{\overline{T_0 W} - \overline{q}}{\overline{W}}. \tag{42}$$

Therefore, under the condition of optimal organization of multiflow heat exchange, the temperature ratio of cold and hot flows at any contact point and the flows' output temperatures must be equal.

In order to define m system's parameters rewrite (32) as follows:

$$\alpha(T_0) = \frac{W(T_0)(\ln T_0 - \ln \overline{T})}{1 - m}. \tag{43}$$

As $\alpha(T_0)$ is non-negative then $T_0 \geq \overline{T}$. Therefore, in a heat exchange system, we should use only the hot flows with a temperature higher than $\overline{T}$. If $T_{01} < \overline{T}$, then all integrals should use $\overline{T}$ as the temperature minimum instead of T_{01} .

When integrating the left and the right-hand sides of (43), where the integral heat exchange efficiency is fixed, we find m

$$m = 1 - \frac{1}{\overline{\alpha}} \int_{T_{01}}^{T_{02}} W(T_0)(\ln T_0 - \ln \overline{T})dT_0. \tag{44}$$

So that the optimal heat exchange efficiency distribution is

$$\alpha(T_0) = \bar{\alpha} \frac{W(T_0)(\ln T_0 - \ln \bar{T})}{\int_{T_{01}}^{T_{02}} W(T_0)(\ln T_0 - \ln \bar{T}) dT_0}, \tag{45}$$

The heat load distribution is

$$q(T_0) = W(T_0)(T_0 - \bar{T}), \tag{46}$$

The EGM is

$$\sigma^* = \bar{\alpha} \frac{(1-m)^2}{m}. \tag{47}$$

When comparing (47) with entropy production $\bar{\sigma}$ in a real working heat exchange system, with a summary heat exchange efficiency $\bar{\alpha}$, the hot flows' input temperatures T_0, their heat capacity $W(T_0)$ and the heating flows' output enthalpy $\overline{W(T_0)T_{\text{out}}(T_0)}$. We can estimate the system's thermodynamic efficiency as $\rho = \frac{\bar{\sigma}^*}{\bar{\sigma}}$.

In order to bring the system's characteristics up to the ideal ones, we should distribute the heating flows and the heat exchange surfaces according to (46), (45) and choose the contact temperatures according to the condition of temperature constancy m (see (44)). We can do it by reducing the heat exchange surface in the heat exchangers with a temperature ratio for the hot and cold flows higher than the system's average value. For the heat exchangers with the temperature ratio lower than the average value, the heat exchange surface should be enlarged. Similarly, the heat intake should be increased for the heating flows with the output temperature higher than the average heating flows' output temperature.

Discreteness of the set of heating flows' temperatures. As a rule, the quantity of heating flows is limited; hence, the temperature set T_0 is discrete. Denote it as T_{i0} and their heat capacities as W_i. All the calculations obtained above remain legitimate, as soon as they were not differentiated in T_0. So, we just need to substitute the integrals with sums over i. Hence

$$\bar{W} = \sum_i W_i, \quad \overline{T_0 W} = \sum_i T_{i0} W_i, \quad \bar{\sigma} = \sum_i \sigma_i(T_{i0}, \alpha(T_{i0}), W_i, q(T_{i0}))$$

etc.

The final formulas for the optimal choice of heated flows' output temperatures, heat load, heat exchange efficiency, the temperature ratio for contact flows and the least possible dissipation take the following form:

$$\left.\begin{aligned}
&\overline{T}=\frac{\sum_i T_{i0}W_i-\overline{q}}{\sum_i W_i},\\
&q^*(T_{i0})=W_i(T_{i0}-\overline{T}),\\
&\alpha^*(T_{i0})=\frac{\overline{\alpha}W_i(\ln T_{i0}-\ln\overline{T})}{\sum_i W_i(\ln T_{i0}-\ln\overline{T})},\\
&m=1-\frac{1}{\overline{\alpha}}\sum_i W_i(\ln T_{i0}-\ln\overline{T}),\\
&\overline{\sigma}^*=\overline{\alpha}\,\frac{(1-m)^2}{m},\\
&\alpha^*(T_{i0})=q^*(T_{i0})=W_i=0,\quad T_{i0}\le\overline{T}.
\end{aligned}\right\}\tag{48}$$

4. An example of thermodynamic efficiency estimation for a heat exchange system

Figure 3 shows a heat exchange system with three hot and three cold flows. The hot flows are denoted by an index i, the cold ones are denoted by an index j. The flows' input and output temperatures expressed in degrees Kelvin for each heat exchanger are displayed at fig. 2. The heat transfer coefficients are expressed in kJ/s. K are displayed inside the circles; heat capacities, with the same dimension, for each input flow are denoted by W_i and W_j.

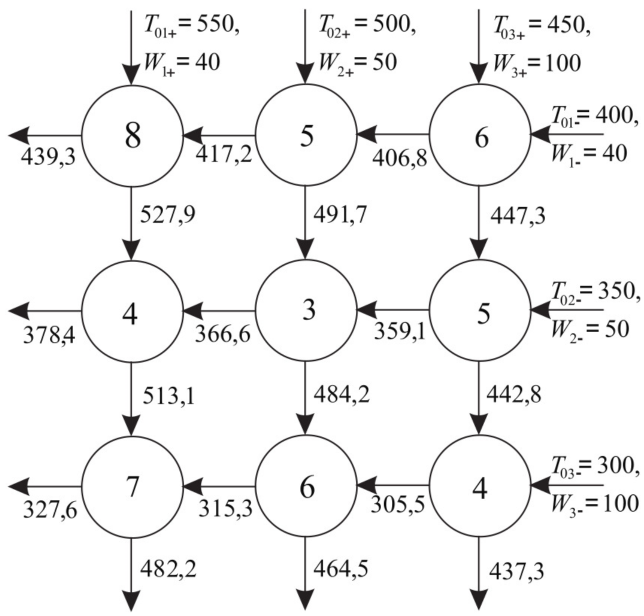

Figure 3. The structure of a multiflow heat exchange system.

Assume that the effective contact temperature for each flow is equal to its average temperature at the exchanger's in- and output. Then there follow the heat exchangers' heat loads q_{ij} expressed in kJ/s and displayed in **Table 1**.

Table 1. The heat exchangers' heat loads.

$j \backslash i$	**1**	**2**	**3**
1	885	416	271.5
2	628.6	375.3	452
3	1290	983.7	549.2

The entropy production for this system is calculated as the sum of all the flows' entropy increment (see (5)).

$$\sigma = \sum_{i=1}^{3} W_i \ln \frac{T_{i\text{out}}}{T_{i0}} + \sum_{j=1}^{3} W_j \ln \frac{T_{j\text{out}}}{T_{j\text{in}}}. \tag{49}$$

This calculation results in $\sigma = 5,574$ kJ/s.K.

Let us use (48) to estimate a thermodynamically optimal heat exchange system where the summary heat load equals $\overline{q}$ =5851 kJ/s and the heat transfer coefficient is equal to $\overline{\alpha}$ =48 kJ/s.K. Using (48), we find the optimal output temperature for the hot flows $\overline{T}$ =457.2 K. When comparing $\overline{T}$ with the hot flows' input temperatures, we come to the conclusion that the third flow with a temperature of 450 K should be excluded from the heat exchange system, and heat exchange surfaces should be redistributed between the first and the second hot flows. Recalculation of $\overline{T}$ for these two hot flows with the same values $\overline{q}$ and $\overline{\alpha}$ results in $\overline{T}$ = 457.2 K. The optimal heat load values taken from the first and the second flows are equal to $\overline{q}(T10)$ = 3712 kJ/s, $\overline{q}(T20)$ = 2140 kJ/s. According to (48), the optimal distribution of the heat exchange surfaces between these two flows gives the following heat exchange coefficients $\overline{\alpha}(T_{10})$ = 29.9 kJ/s.K, $\overline{\alpha}(T_{20})$ = 18.1 kJ/s.K. The effective temperature ratio for the heating flow and the heated one in each heat exchanger should be equal and correspond to m=0.752. The least possible entropy production for this system is σ^*=3.93 kJ/s.K and the thermodynamic efficiency equals $\eta = \frac{\sigma^*}{\sigma} = 0.705$.

The comparison of the optimal heat exchange system with the real one permits refining the real system:

1. By exclusion of the flow with input temperature 450 K out of the system, we enlarge the heat exchange surfaces for the remaining two flows so that the summary heat exchange efficiency increases from 19 to 30 kJ/s.K for the first flow and from 14 to 18 kJ/s.K for the second one.
2. We can distribute the heat exchange area for each flow in such a way that the effective contact temperature ratio of the hot flow and the cold one is equal for each of them and approaches 0.75. Note that in the reference system, the temperature ratio is different for each heat exchanger and varies from 0.63 to 0.88.
3. We recall that the hot flows' output temperatures should approach 457.2 K.

5. Conclusion

In this paper, we define the thermodynamically optimal organization of heat exchange in order to achieve the minimum entropy generation for a system with fixed heat load and fixed summary heat transfer coefficient. We also define the appropriate heat load distribution and heat exchange efficiency distribution for the input flows. Our study permits us to estimate thermodynamic efficiency for a random heat exchange system and to refine it, as well as to analyze the system's dependence on such factors as the input flows' temperature variations or changes in heat exchange surfaces.

Notation

T_+	temperature of the hot heat source, K;
T_-	temperature of a cold heat source, K
T_+^{in}	hot heat source inlet temperature, K;
T_+^{out}	hot heat source outlet temperature, K;
T_-^{in}	cold heat source inlet temperature, K;
T_-^{out}	cold heat source outlet temperature, K;
T_b	boiling point of the stream, K;
l	contact surface, m;
L	total contact surface, m;
Q	thermal load, W;
W_+	heat capacity of a hot stream, W/K;
W_-	heat capacity of a cold stream, W/K;
W_1	heat capacity of the flow entering the mixer, W/K;
W_2	heat capacity of the cold flow leaving the mixer, W/K;
σ	entropy production, W/K;
σ_+	entropy production due to hot flow, W/K;
σ_-	entropy production due to cold flow, W/K;
σ^*	minimum entropy production, W/K;
K	heat transfer coefficient, W/K;
k	specific heat transfer coefficient, W/K;
$z(T_+, T_-)$	temperature multiplier;
r	heat of vaporization, J/mol;
g	flow rate, kg/s.

Indices

i, j	flow indices;
$+, -$	flow indices;
$1, 2$	index of incoming and outgoing flow or heat capacity;
in, out	incoming and outgoing flow index.

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